A MODIFIED DEFINITION OF NTU FOR RECTIFICATION AND ITS RELATION TO NTP

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A new definition is introduced of the number of transfer units which vary less with the operating conditions and composition of the mixture than is the case of the transfer units according to the presently adopted definition. The new definition simplifies the relation between the number of transfer units related to the vapour and the liquid phase as well as the relationship between the number of transfer units and the number of theoretical plates.

The effectiveness of various types of columns for separation processes is usually expressed by the number of theoretical plates (NTP) or the number of transfer units (NTU) per unit height of the column.

The effectiveness expressed in terms of NTP per unit height of the column offers usually the advantage of being usually the quantity less dependent on the operating conditions and composition of the mixture[1] than is the case of NTU. NTP per unit height of a column thus appears to be a more convenient for correlation of experimental data as well as for design and scale-up purposes. On the other hand NTU, which are defined as

$$N_{0g} = \int_{y_1}^{y_2} \frac{\mathrm{d}y}{y^* - y} \quad \text{and} \quad N_{01} = \int_{x_1}^{x_2} \frac{\mathrm{d}x}{x - x^*}, \qquad (1a,b)$$

enable the calculation of the average overall coefficient of mass transfer either to a unit area of interface or the interfacial area in a unit volume of the column.

This offers from the standpoint of fundamental research of the mechanism of transport phenomena a better tool for analysis and eventual comparison with other molecular phenomena, chiefly the heat transfer which is commonly expressed by means of heat transfer coefficients. In addition, the use of NTU appears more adequate than NTP for the description of transport phenomena in packed columns; the latter being more adequate for plate columns.

By introducing a new definition of NTU we can achieve that the new quantity will no longer be so sensitive to the change of operating conditions and composition

of the processed mixture. Moreover, the relationships between NTU related to the vapour and the liquid phase as well as the relations between NTU and NTP considerably simplify.

THEORETICAL

In the derivation of NTU we shall assume that the molar flow rates of the liquid and the vapour phase remain constant throughout the column. The infinitesimal amount of mass of the low boiling component transported by rectification of a binary mixture then may be expressed by

$$dq = G\,dy = -L\,dx\,. \tag{2}$$

Expressing this amount in terms of the vapour and liquid side mass transfer coefficients we obtain

$$dq = K_g(y^* - y)\,dF \tag{3a}$$

and

$$dq = K_1(x - x^*)\,dF\,. \tag{3b}$$

In an analogous manner to that of Vogelpohl[2] we can define the coefficient of mass transfer by

$$dq = K(y^* - y + x - x^*)\,dF\,, \tag{4}$$

where

$$K = K_g K_1/(K_g + K_1)\,. \tag{5}$$

From Eqs (2) and (4) a new definition of the number of transfer units follows in the form

$$N_{0G} = \int_{y_1}^{y_2} \frac{dy}{y^* - y + x - x^*} = KA/G \tag{6a}$$

and

$$N_{0L} = \int_{x_1}^{x_2} \frac{dx}{y^* - y + x - x^*} = KA/L. \tag{6b}$$

Thus

$$N_{0G}/N_{0L} = L/G\,. \tag{7}$$

In order that we may derive the relation between the newly defined NTU and NTP let us consider first the case when the equilibrium curve is a straight line given by

$$(y^* - b)/(x - a) = m \tag{8}$$

and the equation of the operating line is

$$(y - b)/(x - a) = L/G . \tag{9}$$

After substituting from Eq. (8) and (9) into Eq. $(6a)$ and after integration between -y- and -y^*- we obtain

$$N_{0G}/n = m \ln (m\ G/L)/[(m + 1)\ (m\ G/L - 1)] . \tag{10}$$

This equation indicates the number of transfer units per theoretical plate. For the chosen case when m and L/G are constants the number of transfer units per theoretical plate remains also constant. This was the case also with the definition in Eqs $(1a,b)$.

For illustration Fig. 1 plots the dependence of the ratio

$$(N_{0G}/n)/(N_{0G}/n)_{m=L/G}$$

on the ratio $m\ G/L$ for the number of transfer units according to both definitions.

From the figure it is apparent that NTU according to the definition in Eq. $(1a, b)$ vary in the neighbourhood of the value $m\ G/L = 1$ much more markedly than NTU according to the definition in Eq. $(6a)$.

In reality the vapour–liquid equilibrium curves are not straight lines. This means that for a given value of the slope of the operating line, L/G, the value of m, *i.e.* the slope of the equilibrium curve, varies.

The magnitude of this change depends primarily on the shape of the equilibrium curve. Along that section of the equilibrium curve belonging to the operating line of the slope L/G, m usually takes both greater and smaller values than the ratio L/G.
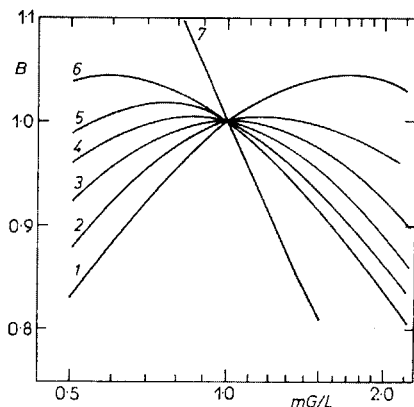

Fig. 1

The Relation between NTP and NTU according to the Two Definitions

B $(N_{0G}/n)/(N_{0G}/n)_{m=L/G}$, 1 $L/G = 0.5$, 2 $L/G = 0.75$, 3 $L/G = 1$, 4 $L/G = 1.25$, 5 $L/G = 1.5$, 6 $L/G = 2$, 7 for the classic NTU (Eq. $(1a)$).

Because the ratio $(N_{0G}/n)/(N_{0G}/n)_{m=G/L}$ does not change significantly in a wide range of the values $m\,G/L$ and because it mostly takes values both larger and greater than the corresponding arc of the equilibrium curve, we may take for the mean value of the ratio (N_{0G}/n) in a number of cases with sufficient accuracy the value at $m = L/G$, i.e. at $m\,G/L = 1$. After substituting for m into Eq. (10) we obtain

$$N_{0G}/n = L/G/(L/G + 1). \tag{11}$$

If the shape of the arc of the equilibrium curve in the corresponding region is not well suited for the above simplification or if the range of m is much too wide, the value of (N_{0G}/n) can be given precision by substituting for m its mean value m taken as

$$\overline{m} = [1/(x_2 - x_1)]\int_{x_1}^{x_2}(\mathrm{d}y^*/\mathrm{d}x)\,\mathrm{d}x = (y_2^* - y_1^*)/(x_2 - x_1). \tag{12}$$

About this value of m the true values vary even less than about (L/G).

DISCUSSION

Despite of the fact that the overall coefficient of mass transfer K was introduced formally by Eq. (4) following from Eqs (3a,b) it can be assigned a physical meaning stemming from the following reasoning. The enrichment of the vapour phase by the low boiling component may be thought to be a consequence of two phenomena: evaporation and condensation.

The evaporation brings into the vapour phase the vapour which is in equilibrium with the liquid and which is thus richer on the low boiling component than the vapour phase alone. On condensation the vapour phase is also enriched by the low boiling component due to the fact that the draining liquid is in equilibrium with the vapour phase and its content of the low boiling component is thus lower than that of the vapour phase. Let us assume that the resulting composition of both phases is the outcome of the above phenomena combined and that Eq. (2) holds in both steps. For the enrichment of the vapour phase during evaporation we may then write

$$G\,\mathrm{d}y_0 = (y^* - y)\,\mathrm{d}Q = -L\,\mathrm{d}x_0, \tag{13}$$

where $\mathrm{d}Q$ designates the number of moles of both components transported from the liquid into the vapour phase.

Similarly for the condensation we may write

$$L\,\mathrm{d}x_k = (x^* - x)\,\mathrm{d}Q = -G\,\mathrm{d}y_k. \tag{14}$$

The net enrichment of the vapour phase on the low boiling component is the sum

$$G\,dy = G\,dy_0 + G dy_k = (y^* - y + x - x^*)\,dQ\,. \tag{15}$$

From Eqs (2), (4) and (15) we thus have

$$K\,dF = dQ\,. \tag{16}$$

The coefficient K in this context represents the number of moles of both components transferred across the unit area of interface from the vapour into the liquid phase and *vice versa*. This can be best illustrated by considering total reflux $(y = x)$ when Eq. $(6a)$ reduces to

$$N_{0G} = \int_{y_1}^{y_2} \frac{dy}{y^* - x^*} \tag{17}$$

and Eq. (15) to

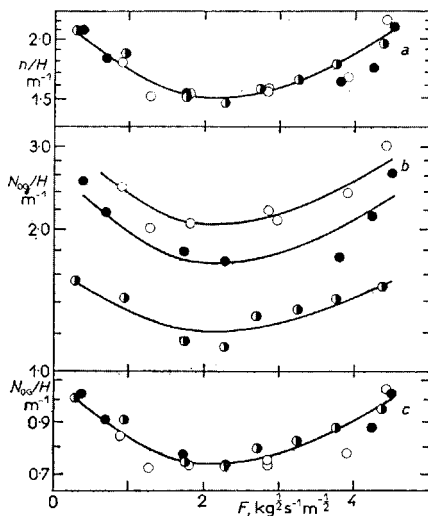$$G\,dy = (y^* - x^*)\,dQ\,. \tag{18}$$

FIG. 2

Separation Ability of Vertical Expanded Metal Sheet Packing in Dependence on Through-Put and Composition of Benzene––Toluene Mixture

○ $x_1 \approx 0.63$, $x_2 \approx 0.96$; ● $x_1 \approx 0.35$, $x_2 \approx 0.90$; ◑ $x_1 \approx 0.05$, $x_2 \approx 0.42$.
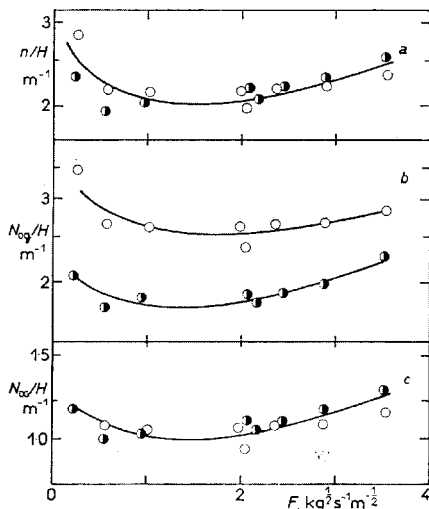
FIG. 3

Separation Ability of Vertical Expanded Metal Sheet Packing in Dependence on Through-Put and Composition of Cyclohexane–n-Heptane Mixture

○ $x_1 \approx 0.80$, $x_2 \approx 0.97$; ◑ $x_1 \approx 0.07$, $x_2 \approx 0.37$.

From the lastequation it is clear that the net enrichment of the vapour phase by the low boiling c omponent is the result of the fact that the vapour phase absorbed dQ moles of the vapour having the mole fraction of the low boiling component y^* and was stripped of dQ moles of liquid of mole fraction of the low boiling component x^*.

The convenience of the newly defined definition of the number of transfer units for expressing the effectiveness of a rectification column is well apparent from Figs 2, 3 and 4 plotting the NTP, the classic NTU and the newly defined NTU per unit height of a column in dependence on the F-factor.

The experimental data used to evaluate the appropriate quantities were obtained during rectification under total reflux of the benzene/toluene and cyclohexane/n-heptane mixture under atmospheric pressure and the ethylbenzene/styrene mixture under 50 Torr pressure. The data were selected from an extensive set of data[3,4] so as to avoid such changes of composition of the rectified mixture leading to substantial changes of physical properties of both phases. From Fig. 2 it is quite apparent that the effectiveness of a column expressed terms of the classic NTU is not a suitable quantity owing to the marked separation of the dependences of the NTU on the F-factor for mixture of various compositions. In case of NTP and the newly
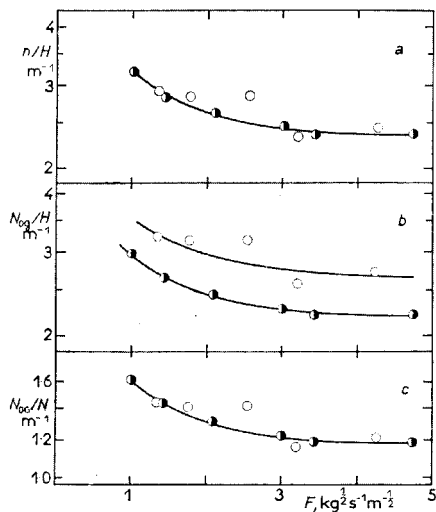

FIG. 4

Separation Ability of Vertical Expanded Metal Sheet Packing in Dependence on Through-Put and Composition of Ethylbenzene–Styrene Mixture at 50 Torr

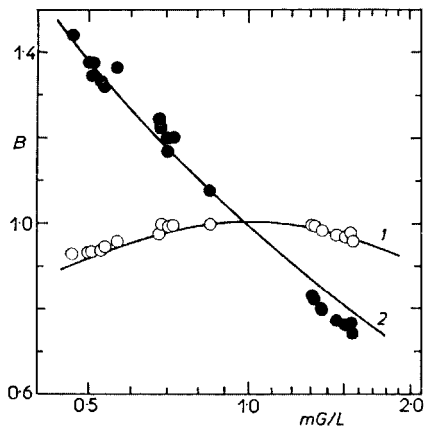○ $x_1 \approx 0.68$, $x_2 \approx 0.91$; ● $x_1 \approx 0.14$, $x_2 \approx 0.41$.

FIG. 5

A Comparsion of Theoretical NTP and NTU according to the Two Definitions with Experimental Data

B $(N_{0G}/n)/(N_{0G}/n)_{m=L/G}$, 1 for the new transfer unit (Eq. (10)) and $L/G = 1$, 2 for the classic transfer unit (Eq. (1a)).

defined NTU the separation does not occur. The significant dependence of the classic NTU has caused that in engineering practice NTP is preferred for expressing the separation ability even of packed columns.

Fig. 5 shows a comparison of the analytical relations between the number of theoretical plates and the number of transfer units both the classic and the newly defined ones with the experimentally found values on the benzene/toluene system. This system was chosen for comparison because it exhibits values of the ratio $m\,G/L$ differing mostly from unity.

The analytical dependences are shown by solid lines; experiments by individual points.

In case of experimental data the ratio $m\,G/L$ was evaluated with the aid of $\bar{m}$ calculated from Eq. *(12)*. The comparison indicates that the data deviate from Eq. *(11)* by 7% at most; this appears in Fig. 5 as a deviation of the points from the straight line $(N_{0G}/n)/(N_{0G}/n)_{m=L/G} = 1$.

The scatter of the points about the curve *1* reflects the deviation caused by using the average value $\bar{m}$ according to Eq. *(12)* in Eq. *(10)*. The deviations do not exceed $\pm 2.5\%$.

LIST OF SYMBOLS

A	interfacial area (m^2)
c, b	coordinates of the intersect of the operating and the equilibrium lines
F	factor defined as $F = u\sqrt{(\varrho)^{1/2}}$ (kg$^{1/2}$s^{-1}m$^{-1/2}$)
G	molar flow rate of vapours (kmol/s)
K	coefficient of mass transfer defined by Eq. *(4)* (kmol/m^2 s)
K_g	coefficient of mass transfer defined by Eq. *(3a)* (kmol/m^2 s)
K_1	coefficient of mass transfer defined by Eq. *(3b)* (kmol/m^2 s)
L	molar flow rate of liquid (kmol/s)
m	slope of equilibrium curve
$N_{0\mathrm{g}}$	overall number of transfer units defined in Eq. *(1a)*
N_{01}	overall number of transfer units defined by Eq. *(1b)*
N_{0G}	overall number of transfer units defined by Eq. *(6a)*
N_{0L}	overall number of transfer units defined by Eq. *(6b)*
n	number of theoretical plates
Q	amount of both components transported across the interface (kmol/s)
q	amount of low boiling component (kmol/s)
u	velocity (m/s)
y	mole fraction of low boiling component in vapour
x	mole fraction of low boiling component in liquid
ϱ	vapour density (kg/m^3)

Indices

1	column bottom
2	column top

o evaporation
k condensation
* equilibrium value

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